Name:

Write your solutions in steps. You need to provide explanations to your answer.

A parametric curve is given by $(\frac{t^3}{3} - t + \frac{2}{3}, t^2 + 2)$.

1. (2.5 points) Find the tangent line of the curve at $(0, 3)$.
2. (2.5 points) Compute the arc length of the curve between $(\frac{2}{3}, 2)$ and $(0, 3)$.
3. (2.5 points) Compute the area bounded between the curve and x-axis on the interval $0 \le x \le \frac{2}{3}$.
4. (2.5 points) Find the point of intersection of this curve with the parametric curve $(t + \frac{1}{3}, t + 2)$.

1. The tangent vector is

$$((\frac{t^3}{3} - t + \frac{2}{3})', (t^2+2)')$$

$$= (t^2 - 1, 2t)$$

The point is at $(0, 3)$ when $t = 1$, so the tangent vector at this point is $(1^2 - 1, 2 \times 1) = (0, 2)$ which implies the tangent line at this point is vertical, so the tangent line is the y-axis. $(x = 0)$.

2. The point $(\frac{2}{3}, 2)$ corresponds to $t = 0$

So the arc length of the curve is

$$\int_0^1 \sqrt{(t^2-1)^2 + (2t)^2}\, dt$$

$$= \int_0^1 t^2 + 1 \, dt = \frac{4}{3}$$

3. $$\int_0^{\frac{2}{3}} y\, dx = \int_1^0 (t^2+2)\, d(\frac{t^3}{3} - t + \frac{2}{3})$$

$$= \int_1^0 (t^2+2)(t^2-1)\, dt$$

$$= \int_0^1 (t^2+2)(1-t^2)\, dt$$

$$= \frac{22}{15}$$

4. $$(\frac{t_1^3}{3} - t_1 + \frac{2}{3}, t_1^2 + 2) = (t_2 - \frac{1}{3}, t_2 + 2)$$

$$\begin{cases} \frac{t_1^3}{3} - t_1 + \frac{2}{3} = t_2 + \frac{1}{3} \\ t_1^2 + 2 = t_2 + 2 \end{cases} \Rightarrow \begin{cases} t_1 = -1 \\ t_2 = 1 \end{cases}$$

So the intersection is $(1 - \frac{1}{3}, 1 + 2) = (\frac{4}{3}, 3)$